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ONE REMARK ON VARIATIONAL PROPERTIES OF GEODESICS IN PSEUDORIEMANNIAN AND GENERALIZED FINSLER SPACES

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Abstract. A new variational property of geodesics in (pseudo-)Riemannian and Finsler spaces has been found.

1. Introduction

Let us assume an n-dimensional **Finsler space** F_n with local coordinates $x \equiv (x^1, \dots, x^n)$ on the underlying manifold M_n, and a (positive definite) metric form with local expression

$$\mathrm{d}s^2 = g_{ij}(x, \dot{x})\mathrm{d}x^i \mathrm{d}x^j \,. \tag{1}$$

Here $g_{ij}(x^1, \dots, x^n, \dot{x}^1, \dots, \dot{x}^n)$ are components of the metric tensor, and $(x, \dot{x})$ denote adapted local coordinates on the tangent bundle TM, i.e., $(\dot{x}^1, \dots, \dot{x}^n)$ are coordinates of the "tangent vector" $\dot{x}$ at x. Metric depends on "positions" and "velocities" in general.

In the Finsler space F_n there exists a (fundamental) function $F(x, \dot{x})$ which is homogeneous of the second degree in $\dot{x}^i$ and satisfies

$$g_{ij}(x, \dot{x}) = \frac{\partial^2 F(x, \dot{x})}{\partial \dot{x}^i \partial \dot{x}^j} \,.$$

Particularly, the equality

$$F(x, \dot{x}) = g_{ij}(x, \dot{x})\mathrm{d}x^i \mathrm{d}x^j$$

holds [3]. As it is well known, in the particular case when components of the metric tensor depend only on position coordinates (i.e., are independent of "velocity coordinates" $\dot{x}$) the Finsler space F_n turns out to be a **Riemannian space** V_n.