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INTEGRAL SUBMANIFOLDS IN THREE-SASAKIAN MANIFOLDS WHOSE MEAN CURVATURE VECTOR FIELDS ARE EIGENVECTORS OF THE LAPLACE OPERATOR

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Abstract. We find the Legendre curves and a class of integral surfaces in a 7-dimensional three-Sasakian manifold whose mean curvature vectors are eigenvectors of the Laplacian or the normal Laplacian and we give the explicit expression for such surfaces in the sphere $\mathbb{S}^7$.

1. Introduction

The class of submanifolds of a (pseudo-) Riemannian manifold, satisfying the condition

$$\Delta H = \lambda H \tag{1}$$

where λ is a constant, H is the mean curvature vector field and Δ denotes the Laplace operator, has been studied by many authors. The study of Euclidean submanifolds with this property was initiated by Chen in [4]. In the same paper biharmonic submanifolds of the Euclidean space are defined as those with harmonic mean curvature vector field.

Results concerning integral submanifolds of a Sasakian manifold of dimension three or five satisfying (1) were obtained in [6, 8, 13, 14]. In some of these papers (see [8, 13, 14]) are studied also the integral curves and surfaces with

$$\Delta^{\perp} H = \lambda H \tag{2}$$

where $\Delta^{\perp}$ is the normal Laplacian.

On the other hand, curves in the 7-sphere endowed with its canonical three-Sasakian structure, which are Legendre curves for all three Sasakian structures, with constant first curvature κ_1 and unit second curvature κ_2, were classified in [1]. It is easy to see that for such curves (1) is verified.