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ON THE KAUP-KUPERSHMIDT EQUATION. COMPLETENESS RELATIONS FOR THE SQUARED SOLUTIONS

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Abstract. We regard a cubic spectral problem associated with the Kaup-Kupershmidt equation. For this spectral problem we prove a completeness of its "squared" solutions and derive the completeness relations which they satisfy. The spectral problem under consideration can be naturally viewed as a $\mathbb{Z}_3$-reduced Zakharov-Shabat problem related to the algebra $\mathfrak{sl}(3,\mathbb{C})$. This observation is crucial for our considerations.

1. Introduction

The **Kaup-Kupershmidt equation** (KKE) is a $1+1$ nonlinear evolution equation given by

$$\partial_t f = \partial^5_{x^5} f + 10 f \partial^3_{x^3} f + 25 \partial_x f \partial^2_{x^2} f + 20 f^2 \partial_x f \tag{1}$$

where $f \in C^\infty(\mathbb{R}^2)$ and ∂_x stands for the partial derivative with respect to the variable x. It is S-integrable, i.e., it possesses a scalar Lax representation $\partial_t \mathcal{L} = [\mathcal{L}, \mathcal{A}]$ with Lax operators of the form

$$\mathcal{L} = \partial^3_{x^3} + 2 f \partial_x + \partial_x f \tag{2}$$

$$\mathcal{A} = 9\partial^5_{x^5} + 30 f \partial^3_{x^3} + 45 \partial_x f \partial^2_{x^2} + (20 f^2 + 35 \partial^2_{x^2} f)\partial_x + 10 \partial^3_{x^3} f + 20 f \partial_x f. \tag{3}$$

It proves to be convenient to work not with scalar but with one-order matrix Lax operators. That is why we factorize the scattering operator $\mathcal{L}$ (see [4])

$$\mathcal{L} = (\partial_x - u)\partial_x(\partial_x + u) \tag{4}$$

where the new function $u(x,t)$ is interrelated with $f(x,t)$ via a Miura transformation as follows

$$f = \partial_x u - \frac{1}{2}u^2. \tag{5}$$