# Two basic Uncertainty Relations in Quantum Mechanics

Andrey Angelow

*Institute of Solid State Physics, Bulgarian Academy of Sciences*
*72 Tzarigradsko chaussee, 1784 Sofia, Bulgaria*

**Abstract.** In the present article, we discuss two types of uncertainty relations in Quantum Mechanics – multiplicative and additive inequalities for two canonical observables. The multiplicative uncertainty relation was discovered by Heisenberg. Few years later (1930) Erwin Schrödinger has generalized and made it more precise than the original. The additive uncertainty relation is based on the three independent statistical moments in Quantum Mechanics – Cov $(q,p)$, Var $(q)$ and Var $(p)$. We discuss the existing symmetry of both types of relations and applicability of the additive form for the estimation of the total error.

**Keywords:** quantum mechanics, foundations of quantum mechanics, uncertainty relations, Schrödinger uncertainty relations
**PACS:** 03.65.-w, 03.65.Ta, 05.30.-d

## INTRODUCTION

It is a fact, that no measurement of any physical quantity could be absolutely determined. The experimentalists know that determination of accuracy needs more time and efforts, than the result of the measurement itself. Each measurement must be completed with two numbers – the value and its uncertainty interval. When the physical quantity is a complex function of several variables it could not be measured directly. In this case we need the additive uncertainty relation in order to define the confidence interval and confidence level in a given experiment. Unlike the classical case, quantum mechanics introduce an additional stochastic correlation between two canonical variables (with commutator $[qp - pq] \neq 0$), which results in multiplicative uncertainty relation.

## SCHRÖDINGER UNCERTAINTY RELATION

The well known Heisenberg uncertainty relation in Quantum Mechanics was precised and generalized by Schrödinger [1, 2] and Robertson [3]. We pay a special attention to Schrödinger's article, where a new therm was taken into account and classes of states with non-vanishing covariance have been shown.

Let us start with the following comparison: on one hand, in the plane we have a simple inequality, on the other in a Hilbert space $\mathscr{H}$ (an infinite-dimension generalization of the plane) we have the Schwarz inequality for every two complex elements $(\phi, \psi \in \mathscr{H})$:

$$1 \geq \frac{(\vec{a}\cdot\vec{b})^2}{|\vec{a}|^2|\vec{b}|^2} = \cos^2\alpha \ \ (n=2) \qquad \leftrightarrow \qquad 1 \geq \frac{|\langle\phi|\psi\rangle|^2}{|\phi|^2|\psi|^2} \ \ (n=\infty). \tag{1}$$

*International Workshop on Complex Structures, Integrability and Vector Fields*
AIP Conf. Proc. 1340, 183-188 (2011); doi: 10.1063/1.3567136