Dedicated to Professor Stancho Dimiev

Initial Value Problem for the Double-Complex Laplace Operator. Eigenvalue Approaches

Lilia N. Apostolova

Institute of Mathematics and Informatics, of the Bulgarian Academy of Sciences
Sofia 1113, Bulgaria

Abstract. Euler formula for the double-complex exponential function is obtained. Eigenfunctions of the zero eigenvalue of the double-complex Laplace operator $\Delta_+ = \frac{\partial^2}{\partial z^2} + i\frac{\partial^2}{\partial w^2}$, where z and w are complex variables, i is the imaginary unit in the complex plane $\mathbb{C}$, are considered. The initial value problem on the Cartesian product of two unit squares in $\mathbb{C}$ for the double-complex Laplace operator is treated by the method of separating of the variables in double-complex context; a double-complex solution is obtained.

Keywords: Double-complex Laplace operator; exponential double-complex function; initial value problem.
PACS: 41.20.Cv, 02.30.Fn

INTRODUCTION

The exponential double-complex function e^{α} is defined by the absolutely convergent power series $\sum_{k\geq 0}\frac{\alpha^k}{k!}$ with the norm $|\alpha|^2 = |z+jw|^2 == |z|^2+|w|^2$, $z,w\in\mathbb{C}$, $i^2=-1$, $j^2 = i$, $j\notin\mathbb{C}$. The following " double-complex Euler formula" is valid:

$$e^{z+jw} = e^z(C(w)+jS(w)) = e^z\left(\cos\frac{(1-i)w}{\sqrt{2}} + j\frac{1+i}{\sqrt{2}}\sin\frac{(1-i)w}{\sqrt{2}}\right)$$

where the even entire function: $C(w) = \cos\frac{(1-i)w}{\sqrt{2}}$ and the odd entire function $S(w) = \frac{1+i}{\sqrt{2}}\sin\frac{(1-i)w}{\sqrt{2}}$ arise.

The eigenfunctions corresponding to the vanishing eigenvalue of the double-complex Laplace operator $\Delta_+ = \frac{\partial^2}{\partial z^2} + i\frac{\partial^2}{\partial w^2}$, are considered. Initial value problem on the Cartesian product of two unit squares in the field of complex numbers $\mathbb{C}$ for the double-complex Laplace operator is treated by the method of separation of the variables in double-complex context. Double-complex periodic solution is obtained.

DOUBLE-COMPLEX LAPLACE OPERATOR

Using the name "double-complex numbers" for the elements of the commutative, associative algebra $\mathbb{C}(1,j)$ (or *DC*) with units $1, j, i, ij$, where $1\in\mathbf{R}$, i is the imaginary unit

International Workshop on Complex Structures, Integrability and Vector Fields
AIP Conf. Proc. 1340, 15-22 (2011); doi: 10.1063/1.3567120