On the Homology Defined by the Electromagnetic Energy Tensor

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Abstract. In the present paper we give a brief study of the observed homology properties of the standard electromagnetic stress-energy-momentum tensor in the important case of null fields, i.e., when the two invariants $I_1 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu}$ and $I_2 = F_{\mu\nu}(*F)^{\mu\nu}$ are vanished. The importance of this special case is based on the fact that this is the only case, when the field and its energy tensor admit unique common null eigen-direction, defining the propagation direction. An additional motivation for the approach presented here comes from the so-called Extended Electrodynamics developed by the authors. In Extended Electrodynamics all nonlinear solutions in the vacuum case have zero invariants. Among the class of nonlinear solutions, there are spatially finite and time-stable spin carrying solutions with photon-like properties.

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INTRODUCTORY REMARKS

From a pure algebraic point of view, we speak about homology (or, cohomology) anytime when we have a linear map D in a vector space $\mathbb{V}$ over a field (e.g. $\mathbb{R}$, or $\mathbb{C}$), or in a module $\mathbb{W}$ over a certain ring, having the property $D \circ D = 0$ [1]. Then we have two related subspaces, $\mathrm{Ker}\,(D) = \{x \in \mathbb{V} : D(x) = 0\}$ and $\mathrm{Im}\,(D) = D(\mathbb{V})$. Since $\mathrm{Im}\,(D)$ is a subspace of $\mathrm{Ker}\,(D)$, we can factorize it, and the corresponding factor space $H(D,\mathbb{V}) = \mathrm{Ker}\,(D)/\mathrm{Im}\,(D)$ is called *homology space* for D. The dual linear map D^* in the dual space $\mathbb{V}^*$ has also the property $D^* \circ D^* = 0$, so we obtain the corresponding *cohomology space* $H^*(D^*,\mathbb{V}^*)$. In this case the map D (resp. D^*) is called *boundary operator* (resp *co-boundary operator*). The elements of $\mathrm{Ker}\,(D)$ (resp. $\mathrm{Ker}\,(D^*)$) are called *cycles* (resp. *co-cycles*), and the elements of $\mathrm{Im}\,(D)$ (resp. $\mathrm{Im}\,(D^*)$) are called *boundaries* (resp. *co-boundaries*).

A basic property of a boundary operator D is that every linear map $\mathscr{B} : \mathbb{V} \to \mathbb{V}$ which commutes with D: $D \circ \mathscr{B} = \mathscr{B} \circ D$, induces a linear map $\mathscr{B}_* : H(D) \to H(D)$. So, a boundary operator realizes the general idea of distinguishing some properties of a class of objects whose properties are important from a certain point of view, and to find those transformations which keep these properties invariant.

A basic example for boundary operator used in theoretical physics is the exterior derivative $\mathbf{d}$ (inducing the de Rham cohomology [2]). This operator acts on the space of differential forms over a manifold, e.g. the Euclidean space $(\mathbb{R}^3, g)$, or the Minkowski space-time $M = (\mathbb{R}^4, \eta)$. The above mentioned basic property of every boundary operator D appears here as a commutation of $\mathbf{d}$ with the smooth maps $\mathfrak{f}$ of the manifold:

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