A Mathematical Base for Fibre Bundle Formulation of Lagrangian Quantum Field Theory

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Abstract. The paper contains a differential-geometric foundations for an attempt to formulate Lagrangian (canonical) quantum field theory on fibre bundles. In it the standard Hilbert space of quantum field theory is replace with a Hilbert bundle; the former playing a role of a (typical) fibre of the letter one. Suitable sections of that bundle replace the ordinary state vectors and the operators on the system's Hilbert space are transformed into morphisms of the same bundle. In particular, the field operators are mapped into corresponding field morphisms.

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INTRODUCTION

The purpose of this work is to be presented grounds for a consistent formulation of quantum field theory in terms of fibre bundles. The ideas for that goal are shared from [1, 2, 3, 4, 5, 6], where the quantum mechanics is formulated on the geometrical language of fibre bundle theory.

We begin with some basic definitions. Special attention is paid on the Hilbert bundles, which will replace the Hilbert spaces of the ordinary quantum field theory, and the metric structure in them. Then it is considered an isomorphism between the fibres of a Hilbert bundle, called the bundle transport. It will play a central role in this investigation. We present a motivation why the (Hilbert) fibre bundles are a natural scene for a mathematical formulation of quantum field theory. At last, some concluding remarks are presented.

FIBRE BUNDLES. HILBERT BUNDLES

To begin with, we present some facts from the theory of fibre bundles [7, 8], in particular the Hilbert ones which will replace the Hilbert spaces in ordinary quantum field theory.

A *bundle* is a triple (E,π,B) of sets E and B, called (total) bundle space and base (space) respectively, and (generally) surjective mapping $\pi\colon E\to B$, called projection. If $b\in B$, $\pi^{-1}(b)$ is the fibre over b and, if $Q\subseteq B$, $(E,\pi,B)|_Q := (\pi^{-1}(Q),\pi|_{\pi^{-1}(Q)},Q)$ is the restriction on Q of a bundle (E,π,B). A *section* of (E,π,B) is a mapping $\sigma\colon B\to E$ such that $\pi\circ\sigma = \mathsf{id}_B$, where id_Z is the identity mapping of a set Z, and their set is

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