A Note on a Homogeneous Structure on an Almost Contact Metric Space

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Abstract. Ambrose and Singer characterized the homogeneity of a Riemannian manifold by the existence of a tensor field T of type $(1,2)$. We consider the homogeneity of odd dimensional Rimemannian manifolds, in particular, that of an almost contact metric space.

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INTRODUCTION

W. Ambrose and I. M. Singer [1] characterized the homogeneity of (M,g) by the existense of some tensor field T of type $(1,2)$ on M, which is called a homogeneous structure.

Theorem 1 ([1]). *Let (M,g) be a connected, simply connected, complete Riemannian manifold. (M,g) is a Riemannian homogeneous space if and only if there exists a tensor field T of type $(1,2)$ on M satisfying*

(1) $g(T(X)Y,Z)+g(Y,T(X)Z)=0$
(2) $\nabla_X R = T(X)\cdot R$
(3) $\nabla_X T = T(X)\cdot T$

where ∇ and R denotes the Riemannian connection and the Riemannian curvature tensor of (M,g), respectively.

Remark 1. *Here, we consider the tensor field $T : \mathfrak{X}(M)\times\mathfrak{X}(M)\to\mathfrak{X}(M)$ of type $(1,2)$ as the map for $X\in\mathfrak{X}(M)$*

$$T(X):\mathfrak{X}(M)\to\mathfrak{X}(M),\quad Y\mapsto T(X)Y$$

and then

$$\begin{aligned}(T(X)\cdot R)(Y,Z)W &:= T(X)(R(Y,Z)W)-R(T(X)Y,Z)W\\ &\quad -R(Y,T(X)Z)W-R(Y,Z)T(X)W\\ (T(X)\cdot T)(Y)Z &:= T(X)T(Y)Z-T(T(X)Y)Z\\ &\quad -T(Y)T(X)Z.\end{aligned}$$

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