Complex Connections on Conformal Kähler Manifolds with Norden Metric

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Abstract. An eight-parametric family of complex connections on a class of complex manifolds with Norden metric is introduced. The form of the curvature tensor with respect to each of these connections is obtained. The conformal group of the considered connections is studied and some conformal invariants are obtained.

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INTRODUCTION

Almost complex manifolds with Norden metric were introduced by A. P. Norden [9] and studied in [4] as generalized *B*-manifolds. A classification of these manifolds with respect to the covariant derivative of the almost complex structure is obtained in [1] and two equivalent classifications were given in [2, 3].

An important problem in the geometry of almost complex manifolds with Norden metric is the study of linear connections preserving the almost complex structure or preserving both, the structure and the metric. The first ones are called almost complex connections, and the second ones are known as natural connections. A special type of natural connection is the canonical one. In [2] it is proved that on an almost complex manifold with Norden metric there exists a unique canonical connection. The canonical connection (called also the *B*-connection) and its conformal group on a conformal Kähler manifold with Norden metric are studied in [3].

In [11] we have obtained a two-parametric family of complex connections on a conformal Kähler manifold with Norden metric and have proved that the curvature tensors corresponding to these connections coincide with the curvature tensors of the canonical connections.

In the present work we continue our research of complex connections on the complex manifolds with Norden metric by focusing our attention on the class of the conformal Kähler manifolds, i.e., manifolds which are conformally equivalent to Käher manifolds with Norden metric. We introduce an eight-parametric family of complex connections on such manifolds and consider their curvature properties. We also study the conformal group of these connections and obtain some conformal invariants. In the last section we give an example of a four-dimensional conformal Kähler manifold with Norden metric, on which the considered complex connections are flat.