On Certain Reductions of Integrable Equations on Symmetric Spaces

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Abstract. We derive a $\mathbb{Z}_2 \times \mathbb{Z}_2$ reduced integrable system which is deeply connected with the famous Heisenberg ferromagnet equation. Its Lax pair is associated with the symmetric space $SO(5)/SO(2) \times SO(3)$. We study the spectral properties of the scattering operator, introduce the basic notions of its direct scattering problem and construct fundamental analytic solutions in terms of Volterra integral equations. We obtain recursion operator to describe the hierarchy of higher order equations associated with the same Lax operator L.

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INTRODUCTION

The main object of study in the present report is a multicomponent equation which generalizes in some sense the Heisenberg ferromagnet equation. The problem of searching generalizations of this classical equation is of current interest, see [8]. Let us recall that the Heisenberg equation can be written in the form

$$iS_t = \frac{1}{2}[S, S_{xx}], \qquad S^2 = \mathbb{1} \tag{1}$$

where $S(x,t)$ is a traceless and antihermitian 2×2 matrix. The elements of $S(x,t)$ can be considered components of a spin vector of an one-dimensional ferromagnet in a closest neighbours approximation. Equation (1) has the following zero curvature representation

$$L(\lambda) = i\partial_x - \lambda S \tag{2}$$

$$A(\lambda) = i\partial_t + \frac{i\lambda}{2}[S, S_x] + 2\lambda^2 S. \tag{3}$$

Equation (1) is the simplest representative of the hierarchy generated by the Lax operator (2) and as it is well known it is gauge equivalent to the nonlinear Schrödinger equation, see for example [5, 13].

The purpose of this work consists in deriving a multicomponent equation whose Lax pair generalizes the pair (2)–(3) and study some of its properties. In this sense this report is a natural continuation of our previous papers [3, 4].

The report is organized as follows. The next three sections contain our main results. In section 2 we obtain a system of 3 nonlinear evolution equations whose Lax pair is related to the symmetric spaces $SO(5)/SO(2) \times SO(3)$. For that purpose we apply the reduction

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