



## EHRENFEST THEOREM IN PRECANONICAL QUANTIZATION\*

IGOR KANATCHIKOV

*School of Physics & Astronomy, University of St Andrews, North Haugh, St Andrews  
KY 16 9SS, Scotland*

**Abstract.** We discuss the precanonical quantization of fields which is based on the De Donder–Weyl (DW) Hamiltonian formulation and does not distinguish between the space and time variables. Classical field equations in DW Hamiltonian form are derived as the equations on the expectation values of the corresponding precanonical quantum operators. This field-theoretic generalization of the quantum mechanical Ehrenfest theorem demonstrates the consistency of three aspects of precanonical field quantization: (i) the precanonical representation of operators in terms of the Clifford (Dirac) algebra valued partial differential operators, (ii) the Dirac-like precanonical generalization of the Schrödinger equation without the distinguished time dimension, and (iii) the definition of the scalar product for calculation of expectation values of operators using the precanonical wave functions.

### 1. Introduction

I am very honoured to contribute a paper to the volume devoted to Professor Jan Ślawianowski. I deeply appreciate his encouraging support during my hard years in Warsaw in the second half of the 1990s. Some aspects of the Ehrenfest theorem in (what I later called) *precanonical quantization* of fields, i.e., a field quantization based on the generalized space-time symmetric Hamiltonian-like formulations known in the calculus of variations of multiple integrals [24, 30], were discussed with him at his Laboratory of Analytical Mechanics and Field Theory already around 1997. Moreover, one of my earlier attempts to understand how a covariant field quantization leading in the classical limit to the corresponding generalized

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