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REDUCTION OF MECHANICAL SYSTEMS ON GROUP MANIFOLDS

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Abstract. We give prescriptions for calculating the dimensions of orbits and isotropy subalgebras in the duals of semidirect products, when one of the algebras is abelian and representation is linear. We are motivated by some mechanical applications.

1. Introduction

We look on equations of motion of certain mechanical systems on group manifolds with symmetry [10, 11], characterized by the property that their Hamiltonian equations of motion can be written down as Lie-Poisson systems. The origins of the Lie-Poisson structure under consideration are due to the Lie group structure of the configuration manifold, the parameters present in the Hamiltonian function and its symmetry properties. The obtained form of the Hamiltonian is called collective function [3] and its dynamics lives on an orbit of the coadjoint representation of semidirect product group. The way of obtaining these equations is one of the methods of reduction [4,6]. The structure of equations provides methods for an analysis of their solutions and integrability. Here we characterize the dimensions of coadjoint orbits and the dimensions of corresponding isotropy subgroups in general and for some physically reasonable systems.

2. The Geometrical Setting

Let the configuration space be the Lie group G, $G = \mathrm{SO}(3, \mathbb{R})$, $G = \mathrm{SL}(3, \mathbb{R})$ or $G = \mathrm{GL}(3, \mathbb{R})$ and let us consider the "rigid" motion on this group meaning the dynamics having the group as its configuration space. The kinetic energy of such