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EULER DECOMPOSITION IN NON-ORTHOGONAL BASES

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Abstract. Here we obtain various covariant expressions for the generalized *Euler* decompositions of three-dimensional rotations and pseudo-rotations based on the vector parameterization developed by *Rodrigues*, *Gibbs* and *Fedorov* [3, 8]. When the chosen rotational axes form a (generally non-orthogonal) basis, the solutions may be written explicitly in terms of the coordinates of the compound vector parameter in this basis. An alternative version of these results is based on considering the entries of the (pseudo-)rotational matrix given by *Rodrigues*' formula. Apart from pure geometry and rigid body mechanics [1, 9, 10], they find applications in areas that vary from robotics and image processing [7], through crystallography and diffractometry [4] to relativity, quantum mechanics and gauge field theories [2,5,6].

1. Vector Algebra in Non-Orthogonal Bases

We remind that if $\{\hat{\mathbf{c}}_k\}$ is a basis in $\mathbb{R}^n$, each vector in this space can be expanded as $\mathbf{x} = x^k\hat{\mathbf{c}}_k$ (summation over repeated upper and lower indices is assumed throughout the text), where the coefficients, or the parallel projections, happen to coincide with the orthogonal projections $x^k = x_k = \hat{\mathbf{c}}_k(\mathbf{x})$ as long as the basis is orthonormal, i.e., $(\hat{\mathbf{c}}_i, \hat{\mathbf{c}}_j) = \delta_{ij}$. In the generic case of non-orthogonal bases, however, there is a crucial difference between upper and lower indices and although the above expansion is still valid, the vector components are equal to the orthogonal