International Conference on Integrability
Recursion Operators and Soliton Interactions
29-31 August 2012, Sofia, Bulgaria
B. Aneva, G. Grahovski
R. Ivanov and D. Mladenov, Eds
Avangard Prima, Sofia 2014, pp 176–187

CLASSICAL AND QUANTUM SYMMETRIES REDUCTION AND INTEGRABILITY*

GIUSEPPE MARMO, GIOVANNI SPARANO† and GAETANO VILASI‡

Dipartimento di Scienze Fisiche, Università degli Studi di Napoli, Istituto Nazionale di Fisica Nucleare Napoli 80126, Italy

† *Dipartimento di Matematica, Università degli Studi di Salerno, Istituto Nazionale di Fisica Nucleare Fisciano 84084, Italy*

‡ *Dipartimento di Fisica, Università degli Studi di Salerno, Istituto Nazionale di Fisica Nucleare Fisciano 84084, Italy*

Abstract. Completely integrable systems always admit more alternative Hamiltonian descriptions. The geometrical formulation of quantum systems shows that similar conclusions hold true also for quantum systems. In addition, the description of quantum systems on Hilbert manifolds, e.g., the complex projective space, shows that not only quantum systems admit alternative Hamiltonian descriptions, they also admit alternative linear descriptions.

1. Introduction

In his *Lectures on Dynamics* [7], Jacobi starts with the problem of integrating the differential equations of motion. He explicitly says: In *Mécanique Analytique one finds everything related to the problem of setting up and transforming the differential equations, but very little on their integration.*

He goes on to elaborate what we nowdays call the Hamilton-Jacobi theory and elaborates on constants of the motion and symmetries.

The aim of our paper is to present a more general point of view in which the Hamilton-Jacobi theory is only an instance of the general procedure of integrating a system by reducing it to a *normal form*. In this respect we follow the view point of Birkhoff, all dynamical systems in the same orbit of the diffeomorphism

*Reprinted from J. Geom. Symmetry Phys. **31** (2013) 105–117.