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PARAMETRIC REPRESENTATIONS OF SO(n) ORTHOGONAL MATRICES FOR THE PURPOSES OF QUADRATIC STABILITY ANALYSIS

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Abstract. The group SO(n) is of a great interest in physics and mechanics because of its numerous applications to problems of monitoring of unknown nonlinear systems. The present paper treats the basic theory of this group for the purposes of quadratic stability analysis of cognitive control systems. It is shown that any transformation of the group SO(n) may be presented as a product of plane transformations in clear analytical forms, appropriate for practical applications. The approach presented here is inspired by the close analogy of plane rotations with the vector-parameterization of the SO(3) group.

1. Introduction

The group SO(n) is a generalization of the SO(3) rotation group acting in $\mathbb{R}^n$. Since $\mathbb{R}^3$ space is the real space where one lives, and where all laws of classical mechanics are valid, the experience with the investigations of the motions in $\mathbb{R}^3$ is helpful to study the motions in higher dimensions. Here is the place to stress the special attention on the group SO(3) since it is a very important in modelling and control of a mechanical system in $\mathbb{R}^3$ and its kinematical description [2]. It is well known that the rigid-body motion in $\mathbb{R}^3$ is described by the Euclidean group E(3), and that the SO(3) group cannot be avoided in the representation of orientations. The appropriate parameterization of SO(3) is one of the most important practical problem in mechanics because it has a great influence over the overall efficiency of all methods [3], [17]. Angular velocity or momentum information is required