International Conference on Integrability
Recursion Operators and Soliton Interactions
29-31 August 2012, Sofia, Bulgaria
B. Aneva, G. Grahovski
R. Ivanov and D. Mladenov, Eds
Avangard Prima, Sofia 2014, pp 225–234

MAPPING BETWEEN NONLINEAR SCHRÖDINGER EQUATIONS WITH REAL AND COMPLEX POTENTIALS*

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Abstract. A mapping between the stationary solutions of nonlinear Schrödinger equations with real and complex potentials is constructed and a set of exact solutions with real energies are obtained for a large class of complex potentials. As specific examples we consider the case of dissipative periodic soliton solutions of the nonlinear Schrödinger equation with complex potential.

1. Introduction

Nonlinear wave phenomena with time evolutions governed by non hermitian Hamiltonians are presently attracting a great interest both from the theoretical and the applicative point of view. The non hermiticity is in general due to the presence of a complex potential in the Hamiltonian accounting for typical dissipative and amplification effects met in classical and quantum contexts [5, 12]. In particular, dissipative solitons [4] of the nonlinear Schrödinger (NLS) equation with periodic complex potentials have been extensively investigated during the past years in connections with the propagation of light in nonlinear optical fibers with periodic modulations of the complex refractive index [13,18]. Recently similar studies were done for matter wave solitons of Bose-Einstein condensates (BEC) trapped in absorbing optical lattices [1, 7] and in the presence of three body interatomic interactions [3]. In the linear context, the recent discovery [6] that the Schrödinger eigenvalue problem with complex potentials that are invariant under the combined parity and time reversal symmetry (so called $\mathcal{PT}$-potentials), may have fully real

*Reprinted from J. Geom. Symmetry Phys. **32** (2013) 25–35.