SPIN COHERENT STATES FOR THE POINCARÉ GROUP

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Abstract

We present here a construction for certain families of coherent states, arising from representations of the Poincaré group, for particles of positive mass and integral or half-integral spin. These coherent states are labelled by points in the classical phase space of the relativistic particle and are associated to affine sections of the Poincaré group, considered as a fibre bundle over the phase space.

1. INTRODUCTION

In the spirit of a general theory, elaborated elsewhere,[1,2] *coherent states* (CS) will be associated in this report to certain *square integrable* representations of groups. The following framework will be adopted: Let G be a locally compact group, $g \mapsto U(g)$ a unitary, irreducible representation (UIR) of G on the (complex, separable) Hilbert space $\mathcal{H}$; let $H \subset G$ be a closed subgroup and suppose that $X = G/H$ carries the *invariant* measure ν. Assume that there exists a (finite) set of vectors η^i, $i = 1, 2, \ldots, n$, in $\mathcal{H}$ and a Borel section $\sigma : X \to G$, such that

$$\sum_{i=1}^{n} \int_X |\eta^i_{\sigma(x)}\rangle\langle\eta^i_{\sigma(x)}| \, d\nu(x) = A, \qquad \eta^i_{\sigma(x)} = U(\sigma(x))\eta^i, \tag{1.1}$$

where A is a bounded positive operator on $\mathcal{H}$, with a densely defined inverse. (The integral in (1.1) is assumed to converge weakly.) We then say that the representation U is *square integrable* mod (H, σ), and call the set of vectors,

$$\mathcal{S} = \{\eta^i_{\sigma(x)} \mid x \in X, \ i = 1, 2, \ldots, n\} \subset \mathcal{H}, \tag{1.2}$$

a *family of covariant coherent states* (CS, for short), for the representation U. If A^{-1} is also a bounded operator, we say that the family of CS $\mathcal{S}$ forms a *rank-n frame*, denoted $\mathcal{F}\{\eta^i_{\sigma(x)}, A, n\}$, and if furthermore, A is a multiple of the identity, we say that the frame is *tight*.

The results presented here generalize some previous work[2,3] on $\mathcal{P}^{\uparrow}_{+}(1,1)$ (the Poincaré group in 1-space and 1-time dimensions) as well as some older work[5] on $\mathcal{P}^{\uparrow}_{+}(1,3)$