

COHERENT STATES AND GLOBAL DIFFERENTIAL GEOMETRY

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Abstract

The relationship between coherent states and geodesics is emphasized. It is found that $CL_0 = \Sigma_0$, where CL_0 is the cut locus of 0 and Σ_0 is the locus of coherent vectors orthogonal to $|0\rangle$. The result is proved for manifolds on which the exponential from the Lie algebra to the Lie group equals the geodesic exponential. The conjugate loci on hermitian symmetric spaces are analyzed also in the context of the coherent state approach. The results are illustrated on the complex Grassmann manifold.

1. INTRODUCTION

Coherent states¹ are an excellent interplay of classical and quantum mechanics.² The local construction of Perelomov's homogeneous coherent states³ has already been globalized, including for Kählerian non-homogeneous manifolds.⁴ On the other hand, the geometric quantization program⁵ yields – at least in principle – a tool towards the quantization program of Dirac on differentiable manifolds. In fact, the coherent state approach offers a straightforward recipe for geometric quantization.⁶ However, there are interesting problems in these two classical fields that have not been attacked yet. One of them is *the relationship between coherent states and geodesics* (see Remark 3 in Ref. 7). The aim of this talk is to explore further this relationship. It is found that for some manifolds there is an intimate connection between *the cut locus* CL_0 ⁸ of a point on the manifold \tilde{M} corresponding to a fixed coherent vector, say $|0\rangle$, and *the polar divisor* Σ_0 , i.e. the locus of coherent vectors orthogonal to $|0\rangle$. Despite the fact that