# NATURAL TRANSFORMATIONS OF LAGRANGIANS INTO $p$-FORMS ON THE TANGENT BUNDLE

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**Abstract**

This paper presents without proofs some theorems giving a complete characterization of natural transformations of finite order of Lagrangians into $p$-forms on the tangent bundle over $n$-dimensional manifolds for $n \geq p+1$ (except for the case $p = 0$, $n = 1$).

## 1. INTRODUCTION

In this paper we will investigate natural transformations of Lagrangians into $p$-forms on the tangent bundle.

The most important examples of these natural transformations are well known in theoretical mechanics. Namely, let $M$ be a differentiable manifold and let $L : TM \longrightarrow \mathbf{R}$ be a smooth function on the tangent bundle. The function $L$ is called Lagrangian. If we denote by $C_M$ the Liouville vector field on $TM$ and by $J_M$ the canonical tangent structure on $TM$, then we can define the following forms on $TM$: the 0-form $E_M(L) = C_M(L) - L$ called energy, the Poincaré-Cartan 1-form $\alpha_M(L) = dL \circ J_M$ and the Poincaré-Cartan 2-form $\omega_M(L) = d\big(\alpha_M(L)\big)$. It is important that $E_M(L)$, $\alpha_M(L)$, $\omega_M(L)$ can be obtained in the same way for an arbitrary manifold $M$ and an arbitrary Lagrangian $L$ and that they are defined without use of a local system of coordinates on $M$. Therefore $E$, $\alpha$, $\omega$ may be regarded as invariants of Lagrangian systems. We will call invariants of this kind the natural transformations of Lagrangians into $p$-forms on the tangent bundle (Definition 1, page 2).

In Ref.2 a complete characterization of natural transformations of finite order of Lagrangians into $p$-forms on the tangent bundle over $n$-dimensional manifolds was given for $p = 0$, 1 and $n \geq p+2$. In Ref.1 a similar characterization was also announced for $p = 2$ and $n \geq p+2$. In this paper we give without proofs a complete classification of these natural transformations for all $p$ and $n \geq p+1$ (except for only one case, namely $p = 0$ and $n = 1$).

The main result of this paper splits into two parts in a natural way.

The first part is the classification of natural transformations in the case $n \geq p+2$. In this case we assert that every natural transformation is a combination of some standard operations such as the Liouville field, exterior differentiation, compositing of a 1-form on the tangent bundle with the canonical tangent structure, form multiplication and addition (Theorem 1, page 5).