$SL(2,\mathbb{R})$-COHERENT STATES AND INTEGRABLE SYSTEMS IN CLASSICAL AND QUANTUM PHYSICS

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1. INTRODUCTION

The group $SL(2,\mathbb{R}) \cong SU(1,1) \cong Sp(1,\mathbb{R})$ appears in various domains of physics as being the *raison d'être* of the integrability of the considered system. We do not here pretend to give an exhaustive list of such systems. We just want to stress the simplifying role played by the complex structure and the related coherent states which naturally appear for some of them. We also intend to clarify the relation existing between those different physical models, their respective complex structures and between two different types of coherent states, namely the $SU(1,1)$ Perelomov coherent states and the $SU(1,1)$ Barut-Girardello ones.

After this conference text was written, the author learned from J-P. Antoine that similar results on integral transforms between different U.I.R realisations of $SL(2,\mathbb{R})$ have recently been presented by D.Basu.[1]

2. TWO IDENTICAL PARTICLES IN ONE DIMENSION

Leinaas and Myrheim exhibit the underlying $SL(2,\mathbb{R})$ symmetry of this system in the following way.

In appropriate units, the relative coordinate and momentum

$$x = x_{(1)} - x_{(2)}, \qquad p = \frac{1}{2}\left(p_{(1)} - p_{(2)}\right) \tag{2.1}$$

of the two particles satisfy the canonical commutation relations, either classical and quantum mechanical

$$\{x,p\} = 1, \qquad [x,p] = i. \tag{2.2}$$

If the particles are identical, x and p do not exist as observables since they are antisymmetric under exchange of particle indices. Observables should be of higher degree and the minimal choice for a basic set is the following:

$$A = \frac{1}{4}\left(p^2 + x^2\right), \qquad B = \frac{1}{4}\left(x^2 - p^2\right), \qquad C = -\frac{1}{4}\left(xp + px\right). \tag{2.3}$$

The Poisson brackets

$$\{A,B\} = C \qquad \{A,C\} = -B, \qquad \{B,C\} = -A \tag{2.4}$$