SYMPLECTIC AND LAGRANGIAN REALIZATION OF POISSON MANIFOLDS

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1. INTRODUCTION

An action principle is usually the starting point to describe physical systems, both particles and fields. A preliminary study usually considers fields as given, i.e. as external fields while particles are thought of as test particles, therefore only the point particle dynamics is dealt with. In this approach the Lagrangian function usually is the sum of three terms: a kinematic term which is quadratic in the velocities, a current-potential coupling term which is linear in the velocities, and a term which depends only on the positions, for instance the electrostatic potential.

The transition to the Hamiltonian description in the symplectic or Poisson formalism allows to absorb the magnetic field in a change of coordinates, so that the momentum $p = \partial\mathcal{L}/\partial\dot{q}$ is replaced by $p + eA$ where A is the vector potential. The electrostatic or other effective potentials appears in the Hamiltonian function.

There are, however, other situations, like the electron-monopole system, where the magnetic field cannot be absorbed in a change of coordinates, this has to do with the fact that the symplectic form in this case is closed but not exact. What we learn from this case is that some external fields, like the magnetic field, will modify the symplectic structure while some other fields will modify the Hamiltonian function. Moreover, to incorporate the magnetic field of the monopole in the Lagrangian picture, one has to add fictitious degrees of freedom, i.e. degrees of freedom which do not carry a dynamical evolution.[1,2]

From the symplectic structure one usually defines Poisson brackets, so that the correspondence with quantum systems is more transparent. As a matter of fact, Poisson brackets arise in the classical limit of quantum systems also for those variables which carry a first order dynamics rather than a second order one, like those arising in the Lagrangian picture, e.g., spin, isospin, color, or other inner variables carried by particles.

We seem to be facing the following situation: when classical mechanics is thought of as limit of quantum mechanics, it seems more natural to deal with the Poisson formalism. If, viceversa, we start with a classical system and try to quantize it, it seems more natural to start with a Lagrangian function to be used either in a Feynman