QUANTUM COHERENT STATES AND THE METHOD OF ORBITS

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Abstract

A general coadjoint orbit of a compact group is quantized by introducing quantum "holomorphic" coordinate functions $\{z_{jk}\}$ on the big cell. Quantum coherent states are defined in a way quite parallel to the classical approach of Perelomov. Any irreducible representation of the deformed enveloping algebra is shown to act in a vector space of polynomials in non-commutative variables $\{z^*_{jk}\}$ according to a simple rule.

1. INTRODUCTION

This note is devoted to compact quantum groups. The goal is to show that, as in the non-deformed case, every irreducible representation τ_λ, with λ a lowest weight, admits an antiholomorphic realization. So this contribution may be considered as an attempt to find a counterpart to the classical Borel-Weil theory. A similar problem for solvable groups has been considered earlier.[1,2]

The construction we are going to describe is done in two steps. First, one has to quantize, as a complex manifold, the coset space of the corresponding group. Here we solve this problem by introducing quantum "holomorphic" coordinate functions $\{z_{jk}\}$ on the big cell. Second, the representation τ_λ is shown to act in a vector space formed by polynomials in non-commutative variables $\{z^*_{jk}\}$, according to a simple rule. In connection with the first step, recently efforts were made to describe deformations of manifolds, particularly to quantize flag and Grassmann manifolds through different approaches.[3–7] We note that our approach encompasses all types of coadjoint orbits. In the second part, the crucial point is the definition of the quantum coherent state which is suggested in a way quite parallel to the classical approach of Perelomov.[8] This may extend in an essential way the range of applications, since the earlier definitions were restricted mostly to the simplest rank-one cases.[9]

This paper highlights and comments on some of the basic steps and results, while the complete proofs will appear elsewhere.[10]