GEOMETRIC COHERENT STATES, MEMBRANES, AND STAR PRODUCTS

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Abstract

An exact star product on symplectic-Kähler manifolds is constructed via quadrangle and hexagon membrane amplitudes. Coherent states and realization of the Dirac axioms over lagrangian submanifolds are described by triangle and pentagon membrane amplitudes. Relations between the star-product quantization and Dirac-type quantization are found.

1. INTRODUCTION

There is an old idea: to quantize classical observables (i.e., functions f on a phase space $\mathcal{X}$) by the integral

$$\mathrm{Op}(f) = \int_{\mathcal{X}} f(x)\mathbf{T}(x)\, dl(x),$$

where dl is a measure on $\mathcal{X}$, and $\mathbf{T}$ is an appropriate family of operators. The nature of this family depends on a concrete situation, system or the type of the quantization theory. The family $\mathbf{T}$ is known under many different names: a Fourier transform of a Lie group representation,[1] a coherent state,[2-4] a reproducing kernel or an overcomplete system,[5-7] a frame,[8] a quantizer.[9] It is important that $\mathbf{T}$ controls a noncommutative "star" product $\circledast$ in the space of classical observables:

$$\mathrm{Op}(f)\cdot \mathrm{Op}(g) = \mathrm{Op}(f \circledast g).$$

The problem is to describe such products $\circledast$ explicitly. An axiomatization of this problem and a search of formulas for $\circledast$ in terms of formal power series in a semiclassical parameter $\hbar$ has led to the famous theory of deformation quantization due to F. Bayen, M. Flato, C. Fronsdal, A. Lichnerowicz, D. Sternheimer[10] (for the existence of such formal products, see Refs.11,12).

There is a crucial open question: how to construct a star product exactly (informally) and geometrically. It would be natural to expect that the global geometry of $\mathcal{X}$

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