INTEGRAL REPRESENTATION OF EIGENFUNCTIONS AND COHERENT STATES FOR THE ZEEMAN EFFECT

Mikhail Karasev and Elena Novikova*

Department of Applied Mathematics
Moscow Institute of Electronics & Mathematics
B.Vuzovsky per., 3/12, Moscow 109028, Russia

Abstract

Coherent states for the hydrogen atom in a magnetic field are constructed via the Bessel functions and Laguerre polynomials. Global integral representations for exact and semiclassical eigenfunctions are obtained using coherent states.

1. INTRODUCTION

We are interested in the following general problem: how to construct eigenfunctions (and eigenvalues) of a quantum system, using information about corresponding classical Hamiltonian system? We study the following integral Ansatz for quantum wave functions

$$\Psi = \int_{\mathcal{L}} \varphi(\beta)\chi_\beta \, d\beta. \tag{1.1}$$

Here $\{\chi_\beta\}$ is a family of vectors in the Hilbert space of a given quantum problem . This family is parametrized by points β running along a classical invariant submanifold $\mathcal{L}$ in the phase space of the problem. The amplitude φ under the integral sign is a certain new wave function. So, we try to transport our quantum problem from the initial Hilbert space to a space of functions over the classical submanifold $\mathcal{L}$. If we choose $\mathcal{L}$ and the family χ in an appropriate way, then the new quantum equation for the amplitude φ might be global over $\mathcal{L}$ and simpler than the initial problem. For instance, we can easily solve this equation in a semiclassical approximation.

Of course, this is only a general idea. For details, applications and a geometrical interpretation of the family χ and the equation for the amplitude φ, we refer to a series of works[1-3] begun in 1989 (for further developments, see Refs. 4-7).

Concerning the relations to the geometric quantization procedure, to the Bargmann representation and the coherent state theory, see Refs. 7-10.

In particular, there is an interesting case when the submanifold $\mathcal{L}$ is a Liouville torus of an integrable Hamiltonian system. In this case, there is a basis of first integrals

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