EXPLICITLY COVARIANT ALGEBRAIC REPRESENTATIONS FOR TRANSITIONAL CURRENTS OF SPIN-1/2 PARTICLES

M. I. Krivoruchenko

Institute for Theoretical and Experimental Physics
ITEP, 117259, Moscow, Russia

Abstract

In non-relativistic quantum mechanics and in relativistic quantum theory, there exist explicitly covariant algebraic representations for transitional currents of massive spin-1/2 particles. We show that in the massless case such representations exist only for transitional currents which are non-diagonal in helicity. The diagonal currents have algebraic representations which are not explicitly covariant.

1. INTRODUCTION

In non-relativistic quantum mechanics, the transitional currents $\chi^+(\zeta')\chi(\zeta)$ or $\chi^+(\zeta')\boldsymbol{\sigma}\chi(\zeta)$, where $\boldsymbol{\sigma}$ are the Pauli matrices and $\chi(\zeta'), \chi(\zeta)$ are spinors describing particles with polarizations ζ', ζ, determine the amplitudes of processes involving spin-1/2 particles. The cross-sections are usually computed by squaring the absolute values of the amplitudes. The spinors are then removed with the use of the technique of projection operators.[1] In this way one obtains for the cross-sections algebraic expressions which depend on the vectors ζ', ζ in an explicitly covariant form with respect to the three-dimensional rotations.

An analogous prescription is used in the relativistic quantum theory. Cross-sections and decay probabilities contain currents defined through the Dirac bispinors. The explicit dependence on momenta and particle polarization vectors remains hidden in the currents of this type. Just as in non-relativistic quantum mechanics, the bispinors are removed in a covariant way from the squares of absolute values of the matrix elements.[2-3]

In the case of a complicated tensor structure, it seems easier to deal with the algebraic expressions for the amplitudes rather than with transition probabilities. For example, the Compton effect amplitude is a contraction of a rank-two tensor with polarization vectors of two photons. The cross-section of this process is quadratic in the amplitude and requires already a rank-four tensor for its determination.

The explicitly covariant algebraic representations for transitional currents can be constructed in non-relativistic quantum mechanics and in the relativistic quantum theory of massive particles.[4-6]

In the next section, we discuss these representations and apply them for the analysis of the problem of electron scattering in an external Coulomb field. In Sect. 3, we discuss