ON GEOMETRIC METHODS IN THE DESCRIPTION OF QUANTUM FLUIDS

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Abstract

Some geometric ideas concerning the description of superfluid helium are presented. Results of application of knot theory to dense systems of quantum vortices are shown. Suggestions concerning applications of Kähler geometry in the description of superfluid helium by means of coadjont orbits of volume preserving diffemorphism groups are made.

This paper is devoted to the presentation of some recent geometric ideas in the description of superfluid helium. It is divided into two parts. In the first part, we show some results obtained by applying knot theory to dense systems of vortices in superfluid helium. In the second part, we present some ideas as to when and how one can introduce Kähler geometry in the description of coadjoint orbits of volume preserving diffeomorphism groups.

1. KNOT THEORY IN THE DESCRIPTION OF SUPERFLUID HELIUM

The model of vortices used previously[1] considers them as solutions of equations for a complex scalar field coupled to a $U(1)$ gauge field. The gauge field is concentrated in a very narrow region of the vortex core. Therefore, the effective description of rarely distributed vortices is in terms of the scalar field alone and the effective description of dense systems of vortices is in terms of the $U(1)$ gauge field $A(\bar{x},t) = A_\mu dx^\mu$. Before formulating the problem of how to find a proper action for such a system, let me recall some historical background.

In the 1950's Feynman suggested that the superfluid phase transition is caused by proliferating vortex rings. However, many models based on this picture were not successful in obtaining proper characteristics of the phase transition. In contrast, RGT techniques, which do not use such a picture, proved more powerful.[2] Nevertheless, recent experiments performed by the group of Prof. Mc Clintock, showed that very dense systems of vortices are present in superfluid helium close to the critical point.[3] This observation is a strong support for the Feynman picture. On the other hand, we know that in such dense systems of vortices there should occur processes of reconnection, which lead to the appearance of complicated linked and knotted vortex structures in