GALACTIC DYNAMICS IN THE SIEGEL HALF-PLANE

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Abstract

The dynamics of rotating galaxies is modeled by a Hamiltonian Lax system for which the phase space is a homogeneous G-manifold with the Lie group G equal to either the noncompact real symplectic group $Sp(n, R)$ or a maximal parabolic subgroup $GCM(n)$. The dimensions $n = 1, 2, 3$ correspond respectively to breathing mode oscillations, planar rotations, and three-dimensional collective motion. The homogeneous $GCM(3)$-manifolds correspond to the Riemann ellipsoids. The homogeneous G-manifold $Sp(n, R)/U(n)$, where $U(n)$ is the maximal compact subgroup, is a classical complex domain diffeomorphic to the Siegel upper half-plane S_n. Equilibrium galactic radii are determined for S_1 systems.

1. MANY-BODY COLLECTIVE DYNAMICS

The mathematical description of galactic dynamics is, in principle, straightforward. The phase space for a galaxy of A stars is $\mathbf{R}^{6A}$, and the Hamiltonian is the sum of the kinetic energy of each star, $T = \sum_\alpha p_\alpha^2/2m_\alpha$, plus the gravitational potential energy, $V = -G\sum_{\alpha<\beta} m_\alpha m_\beta/|\mathbf{x}_\alpha - \mathbf{x}_\beta|$, where the sums are over the particle index $\alpha, \beta = 1, \ldots, A$, m_α

denotes the mass of star α, and $\mathbf{x}_\alpha$, $\mathbf{p}_\alpha$ denote the Cartesian position and momentum vectors for star α. The time evolution of the galaxy $(\mathbf{x}_\alpha(t), \mathbf{p}_\alpha(t)) \in \mathbf{R}^{6A}$ is computed by integrating the Hamiltonian vectorfield given the initial galactic state $(\mathbf{x}_\alpha(0), \mathbf{p}_\alpha(0))$ in phase space.

There are two practical problems with this analysis because the number of stars is, in a word, astronomical, $A \sim 10^{11} - 10^{14}$. First, the computation of the integral curves is intractable even using the fastest parallel processing supercomputer. Second, the state of the system in $\mathbf{R}^{6A}$ is not observationally measurable. Astronomical observations provide only partial information about average properties of the stellar distributions in position and momentum space. For example, the projection of the axes lengths of an elliptical galaxy upon the two-dimensional plane perpendicular to the line of sight can be observed optically, and the dispersion in the stellar velocity fields can be determined from the doppler broadening of galactic spectral lines. Thus, our knowledge of real galactic states is very crude, and, even if the initial states could be measured, we cannot calculate their time evolutions.

These technical limitations suggest an alternative analysis that focuses upon the observables which, in fact, are really measurable, and ignores the degrees of freedom