PREQUANTIZATION

D. J. Simms

School of Mathematics, Trinity College
Dublin 2, Ireland

Abstract

We give an exposition of the concept of prequantization, which enables standard constructions for cotangent bundles to be extended to symplectic manifolds whose symplectic forms represents an integral de Rham cohomology class.

1. INTRODUCTION

The concept of geometric quantization as developed by Kostant[1] and Souriau[2] is now thirty years old. Since it is based on the symplectic structure of phase space, rather than the structure of the cotangent bundle of configuration space, it has gained in relevance over the years, as symplectic manifolds have arisen as moduli spaces.

The starting point of geometric quantization is the concept of prequantization which is the most fundamental point of contact between the classical mechanical notions of Hamiltonian vector fields and classical Poisson brackets on one side, and linear operators and operator commutators on the other.

For a cotangent bundle the associated symplectic form $\omega = \sum dp_i \wedge dq_i$ is exact with $\omega = d\alpha$ where α is the canonical 1-form, and the action S is the integral of the 1-form $\alpha - Hdt$.

Prequantization gives a method of considering Dirac amplitude $\exp iS$ for the wider class of symplectic manifolds for which ω is not necessarily exact, but does represent an integral de Rham cohomology class.

More recently a particular impetus to these ideas has been given by Witten's treatment of the Jones polynomial. This gives rise to the moduli space of flat G-bundles over a a compact surface of fixed genus, for G a compact simple Lie group. The moduli space in this case has been shown by Atiyah and Bott to be a symplectic manifold whose symplectic form represents an integral de Rham cohomology class. For a recent relevant article and bibliography see J.-L. Brylinski and D. McLaughlin.[3]

2. THE LINE BUNDLE

Let M be a symplectic manifold with symplectic form ω. Thus ω is a closed 2-form, $d\omega = 0$, and nondegenerate.