GRADED CONTRACTIONS OF $so(4,2)$

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Abstract

All $\mathbb{Z}_2 \times \mathbb{Z}_2$-graded contractions preserving the space isotropy with grading induced by the Π (space inversion) and Θ (time reversal) automorphisms of the Lie algebra $so(4,2)$ are listed. Some properties of these contractions are discussed.

1. INTRODUCTION

$\mathbb{Z}_2 \times \mathbb{Z}_2$-graded contractions of the kinematical groups of space-time were the subject of a previous article.[1] There the new method[2] based on the preservation of a grading through the contraction was studied in the case of the complex Lie algebra B_2 and its real forms. These contractions are physically interesting because the non–compact real forms $o(4,1)$ and $o(3,2)$ correspond to the de Sitter and the anti–de Sitter groups respectively, and among the contracted Lie algebras those of the Poincaré and the Galilei group are present. As a result, the classification of the 10-dimensional kinematical groups emerged under the very natural assumptions of space isotropy and preservation of the $\mathbb{Z}_2 \times \mathbb{Z}_2$-grading induced by the Lie algebra automorphisms of space inversion Π and time reversal Θ (see also Refs. 3,4).

We find it worthwhile and also interesting from the physical point of view to extend the scope of our investigation to the *conformal group of space-time* because of its overall importance, especially in quantum field theories as the symmetry of theories of massless particles.[5] As a matter of fact, already I. E. Segal[6] gave the very first definition of a contraction yielding an approximate (asymptotic) symmetry from the initial exact symmetry on an example of the conformal Lie algebra $so(4,2)$.

In order that the results of this paper could be compared with the preceding ones,[1] we dwell on the assumptions of the space isotropy and of preservation of the $\Pi \times \Theta$–grading.