ON DIRAC TYPE BRACKETS

Yurii M. Vorobjev[1*] and Ruben Flores Espinoza[2*]

[1] Department of Applied Mathematics
Moscow Institute of Electronics & Mathematics
B.Vuzovsky per., 3/12, Moscow 109028, Russia
[2] Department of Mathematics, University of Sonora
Rosales y Transversal, 83000 Hermosillo, Sonora, Mexico

Abstract

We investigate the class of Poisson structures with a transversally maximal Lie algebra of infinitesimal automorphisms. We describe such Poisson structures in terms of singular 2-forms and in terms of some universal de Rham cohomology classes of symplectic leaves.

1. INTRODUCTION

We consider a class of *regular, degenerate Poisson* structures with *transversally* maximal Lie algebra of infinitesimal automorphisms (or Poisson vector fields). This class naturally arises in the deformation and cohomology theory of Poisson brackets[1−8] and includes, for example, the *Dirac bracket* and some of its generalization.[7,9]

Let M be a symplectic manifold and $A^1, \dots, A^r \in C^\infty(M)$ be a set of independent functions such that the matrix $\Delta = ((\Delta^{ij})) \equiv ((\{A^i, A^j\}))$ of pairwise Poisson brackets on M is nondegenerate everywhere. Then the standard Dirac bracket on M is given by the formula

$$\{f,g\}_{\text{DIR}} = \{f,g\} + \sum_{1\leq i,j\leq r} \Delta_{ij}\{A^i, f\}\{A^j, g\}, \tag{1.1}$$

where $\Delta_{is}\Delta^{sj} = \delta_i^j$. It is clear that the functions A^j are the *Casimir* functions relative to (1.1) and the corresponding symplectic leaves Ω coincide with the level sets of these functions. Consider the set of independent vector fields on M

$$z_i = \sum_{1\leq j\leq r} \Delta_{ij}\, \mathrm{ad}(A^j), \qquad i = 1, \dots, r, \tag{1.2}$$

where $\mathrm{ad}(A^j)$ is the *Hamiltonian field* of A^j with respect to the original Poisson structure on M. We claim[4,7,8] that $z_1, \dots, z_r$ are infinitesimal automorphisms of (1.1), *transverse* to symplectic leaves Ω at each point.

*Research partially supported by Conacyt grant 489100-5-4106 E and Russian Foundation for Fundamental Research