QUANTUM TRIGONOMETRY AND PHASE-SPACE PROPENSITY

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Abstract

Quantum trigonometry, corresponding to the operational detection of the quantum phase of an optical field, is derived from the phase-space propensity.

1. INTRODUCTION

The description of quantum phase fluctuations of optical fields is a nontrivial problem because of the well known difficulties associated with the definition of a Hermitian phase operator.[1,2] In the published literature one can find several competing descriptions of quantum phase fluctuations based on different assumptions and mathematical constructions of the phase operator. Only in the classical limit, i.e., when the electromagnetic fields are strong (have a large number of photons), a semiclassical or a statistical description of phase fluctuations is possible. In order to avoid the difficulty with the definition of the hermitian phase operator Susskind and Glogower[3] introduced two operators S and C corresponding to the classical quantities $\sin\varphi$ and $\cos\varphi$:

$$C = \frac{1}{2}\left(\frac{1}{\sqrt{n+1}}b + b^\dagger\frac{1}{\sqrt{n+1}}\right), \tag{1.1}$$

$$S = \frac{1}{2i}\left(\frac{1}{\sqrt{n+1}}b - b^\dagger\frac{1}{\sqrt{n+1}}\right), \tag{1.2}$$

where b and $b^\dagger$ are the annihilation and creation operators of the single mode and $n = b^\dagger b$ is the photon number operator. However these operators do not commute and the trigonometric unity rule does not hold, it is violated for the vacuum state in the form $(S^2 + C^2)|0\rangle = \frac{1}{2}|0\rangle$. Many new phase operators have been suggested as summarized in the excellent review by Carruthers and Nieto.[4]

It is the purpose of this paper to investigate the quantum trigonometry of a single mode of a harmonic oscillator, using the concept of the phase-space propensity and the associated operational positive operator valued measure (POVM). Our approach