SEMICLASSICAL BEHAVIOUR OF COHERENT STATES ON THE CIRCLE

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ABSTRACT

Families of coherent states labeled by points in T^*S^1 are constructed in $L^2(S^1)$ by decomposition of the Weyl–Heisenberg coherent states in $L^2(R)$. This same decomposition also gives a proper definition of the Weyl correspondence on $S^1 \times R$. Coherent states in $L^2(S^1)$ are shown to provide a realization of $L^2(S^1)$ by entire functions (similar to the standard Fock–Bargmann construction). Their semi-classical behaviour is shown to be as expected: they concentrate, as $\hbar \to 0$, on the phase space point by which they are labeled.

1. Introduction

We consider a system having S^1 as configuration space. Its phase space is $T^*S^1 \equiv S^1 \times \mathbf{R}$ and its quantum mechanical Hilbert space $L^2(S^1)$. We address some questions concerning quantization (symbol calculus) and the classical limit in this context.

In section 2 we construct coherent states $|q,p;k\rangle$ in $L^2(S^1)$, labeled by points (q,p) in T^*S^1 (here $k \in [0, 2\pi/a)$, if we identify S^1 with $[0,a)$). We construct the associated Fock–Bargmann space of entire functions on T^*S^1 and explain how this space could also have been obtained by geometric quantization. In order to study the semi-classical behaviour of the $|q,p;k\rangle$, we introduce in section 3 what we think to be a natural notion of Weyl quantization on T^*S^1. We then show (in section 4) that, if $\hat{f}^{(k)}$ is the Weyl quantization of the phase space function f on T^*S^1, then

$$\lim_{\hbar\to 0} \langle q,p;k|\hat{f}^{(k)}|q,p;k\rangle = f(q,p), \tag{1.1}$$

$$\lim_{\hbar\to 0} \langle q,p;k|\frac{[\hat{f}^{(k)},\hat{g}^{(k)}]}{i\hbar}|q,p;k\rangle = \{f,g\}(q,p). \tag{1.2}$$

Equation (1.1) shows that the coherent states $|q,p;k\rangle$ concentrate in phase space around the point (q,p), while (1.2) says that the Wick symbol of $[\hat{f}^{(k)},\hat{g}^{(k)}]$ has the expected leading term behaviour.

2. Coherent States and a "Fock–Bargmann" Representation for $L^2(S^1)$

2.1. Coherent States

In this section we are going to show that families of coherent states (CS, from now on) in $L^2(S^1)$ can be constructed by decomposition of the standard Weyl–Heisenberg CS in $L^2(\mathbf{R})$ [1]. The