

CONNECTIONS AND EXCITED WAVEPACKETS  
OVER INVARIANT ISOTROPIC TORUS

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ABSTRACT

It is shown how to construct a semiclassical wave function by means of transport of certain universal packets (or wavelets) along an individual invariant tori of a Hamilton system using a connection generated by infinitesimal integrals of motion.

## 1. Introduction

Let a Hamiltonian system corresponding to the function  $H(q, p)$  on  $\mathbb{R}^{2d}$  have an invariant isotropic torus  $\Lambda \approx \mathbb{T}^k$  lying at the level of constant energy  $\{H = \lambda\}$ . What can we say in this case about the spectrum and eigenfunctions of the quantum operator  $\hat{H} = H(q, -i\hbar\partial/\partial q)$ , at least in a semiclassical approximation as  $\hbar \rightarrow 0$ ? (We define the operator  $\hat{H}$  by means of the Weyl symmetrization of  $q$  and  $-i\hbar\partial/\partial q$  and assume that at infinity all the derivatives of the symbol  $H$  grow not greater than a certain polynomial).

There is the following *a priori* hypothesis: one can find a set of numbers  $\lambda_{m,n}$  close to  $\lambda$ , and of functions  $\psi_{m,n}(q)$  whose oscillation front is close to  $\Lambda$ , so that the pair  $\lambda_{m,n}, \psi_{m,n}$  approximates the exact spectral data of the operator  $\hat{H}$  with a precision  $o(\hbar)$ . Here the indices  $m, n$  denote the quantum numbers responsible for the excitation of the torus along the "action variables" and the "oscillator variables" skew-orthogonal to  $\Lambda$ .

Of course, this hypothesis can be realized only if a number of additional conditions is satisfied. Usually, in this problem, a classical method for matching the local WKB-asymptotics and their Fourier transformations is used (Keller, Maslov, Hermander, Babich and others; for details and references, see <sup>1-5</sup>). Here we want to describe a simple global construction for quasi-modes  $\psi_{m,n}$  by means of another technique proposed in <sup>6-8</sup>, which does not use gluing, matching, etc.