CONNECTIONS AND EXCITED WAVEPACKETS OVER INVARIANT ISOTROPIC TORUS

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ABSTRACT

It is shown how to construct a semiclassical wave function by means of transport of certain universal packets (or wavelets) along an individual invariant tori of a Hamilton system using a connection generated by infinitesimal integrals of motion.

1. Introduction

Let a Hamiltonian system corresponding to the function $H(q,p)$ on $\mathbb{R}^{2d}$ have an invariant isotropic torus $\Lambda \approx \mathbb{T}^k$ lying at the level of constant energy $\{H = \lambda\}$. What can we say in this case about the spectrum and eigenfunctions of the quantum operator $\widehat{H} = H(q, -i\hbar\partial/\partial q)$, at least in a semiclassical approximation as $\hbar \to 0$? (We define the operator $\widehat{H}$ by means of the Weyl symmetrization of q and $-i\hbar\partial/\partial q$ and assume that at infinity all the derivatives of the symbol H grow not greater than a certain polynomial).

There is the following *à priori* hypothesis: one can find a set of numbers $\lambda_{m,n}$ close to λ, and of functions $\psi_{m,n}(q)$ whose oscillation front is close to Λ, so that the pair $\lambda_{m,n}$, $\psi_{m,n}$ approximates the exact spectral data of the operator $\widehat{H}$ with a precision $o(\hbar)$. Here the indices m, n denote the quantum numbers responsible for the excitation of the torus along the "action variables" and the "oscillator variables" skew-orthogonal to Λ.

Of course, this hypothesis can be realized only if a number of additional conditions is satisfied. Usually, in this problem, a classical method for matching the local WKB-asymptotics and their Fourier transformations is used (Keller, Maslov, Hermander, Babich and others; for details and references, see [1–5]). Here we want to describe a simple global construction for quasi-modes $\psi_{m,n}$ by means of another technique proposed in [6–8], which does not use gluing, matching, etc.