A CONSTRUCTION OF A STAR PRODUCT ON A POISSON MANIFOLD

HIDEKI OMORI
Dept. of Math. Fac. of Science and Technology
Science Univ. of Tokyo, Noda, Chiba 278, Japan

and

YOSHIAKI MAEDA
Dept. of Math. Fac. of Science and Technology,
Keio Univ., Hiyoshi, Yokohama 223, Japan

and

AKIRA YOSHIOKA
Dept. of Math. Fac. of Science and Technology,
Science Univ. of Tokyo, Noda, Chiba 278, Japan

ABSTRACT

An inductive method is given for constructing a star product on a Poisson manifold. The necessary and sufficient conditions are presented for this method to be successful.

Introduction

Let M be a C^∞ manifold with a Poisson structure $\{\ ,\ \}$ and let $\mathbf{a} = C^\infty(M)$ be the set of C-valued C^∞ functions on M. $\mathbf{a}$ is a commutative topological algebra with the C^∞ topology. A Poisson structure $\{\ ,\ \}$ is a bilinear mapping $\{\ ,\ \} : \mathbf{a} \times \mathbf{a} \longrightarrow \mathbf{a}$ with which $\mathbf{a}$ is a Lie algebra and satisfies the derivation property[7].

$$\{f, gh\} = \{f, g\}h + g\{f, h\} \text{ for any } f, g, h \in \mathbf{a}. \tag{1.1}$$

$(\mathbf{a}, \{,\})$ is called a *Poisson algebra* .

With a formal parameter ν, we set the direct product

$$\mathbf{a}[[\nu]] = \Pi_{n=0}^{\infty} \nu^n \mathbf{a}. \tag{1.2}$$

Let us consider a bilinear product $* : \mathbf{a}[[\nu]] \times \mathbf{a}[[\nu]] \to \mathbf{a}[[\nu]]$, which is written as

$$f * g = \sum_{n=0}^{\infty} \nu^n \pi_n(f, g),\ \pi_n(f, g) \in \mathbf{a}, \text{ for any } f, g \in \mathbf{a}, \tag{1.3}$$