RIEMANNIAN AND SUPERSYMMETRIC PROPERTIES OF SQUEEZED AND CORRELATED STATES

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ABSTRACT

The Riemannian metrics of Provost-Vallée and the Onofri' Kaehler metrics are constructed and discussed for the set (and some subsets) of squeezed and correlated states (SCS) that obey the Schrödinger-Robertson uncertainty relation. The differences between the geometric properties of the correlated and noncorrelated squeezed states are pointed out. The simple relationship between SCS and the $Osp(1/2, R)$ supercoherent states is established, the latters being a linear combination (with Grassman variable coefficients) of squeezed ground and one photon states.

1. Introduction

The promising applications of squeezed and correlated states (SCS) (see for example[1,2]) stimulate the further investigations of their mathematical and physical properties. Unless otherwise stated in this paper we consider SCS as Schrödinger minimum uncertainty states (SMUS)[3], i.e. as states which minimize the Schrödinger-Robertson uncertainty relation[4]

$$\sigma_q^2\sigma_p^2 \geq \frac{1}{4}(1+4c^2), \tag{1}$$

where σ_q, σ_p and c are the second momenta of the quadrature operators Q and P ($[Q,P]=i$),

$$\sigma_A = \langle A^2\rangle - \langle A\rangle^2,\ A = Q, P,\ c = \frac{1}{2}\langle QP+PQ\rangle - \langle Q\rangle\langle P\rangle.$$

They are equivalent[3] to the famous Stoler states $| z;\alpha\rangle$[5] (known also as squeezed states or squeezed minimum uncertainty states[6], although for pure imaginary z neither σ_q nor σ_p is less than the ground state value 1/2) and to the two-photon coherent states[7].

From the group theoretical point of view SMUS are equivalent[3] to the group-related coherent states (G-CS) with maximal symmetry[8], the group G in this case being the (nonsolvable, nonsemisimple) semidirect product $H_w \wedge SU(1,1)$ of the Heizenberg-Weyl group H_w and the quasi-unitary group $SU(1,1) \sim Sp(2,R)$. The representation involved is generated by the semidirect sum Lie algebra (the two-photon algebra)

$$\begin{aligned} h_w =&\ \text{lin. env. } \{1, a, a^\dagger\}, \\ su(1,1) =&\ \text{lin. env. } \{K_- = \frac{1}{2}a^2,\ K_+ = \frac{1}{2}(a^\dagger)^2,\ K_0 = \frac{1}{2}(a^\dagger a + \frac{1}{2})\}, \end{aligned} \tag{2}$$