QUANTUM FRAMES, QUANTIZATION AND DEQUANTIZATION

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Abstract

A continuous frame in a Hilbert space is a concept well adapted for constructing very general classes of coherent states, in particular those associated to group representations which are square integrable only on a homogeneous space. In addition, (quantum) frames provide a method of quantization which generalizes the coherent state approach and fits in neatly with the operational meaning of quantum measurements. We discuss this approach in detail, taking as our working example the case of the Poincaré group in 1+1 space-time dimensions. We also compare this approach to the familiar geometric quantization method, which turns out to be less versatile than the new one.

1. WHAT IS A QUANTUM FRAME ?

Reference frames are vital in classical physics, from the early perception of the world by children to the learning of elementary physics, all the way to general relativity. In elementary mechanics, a frame is simply a basis $\{e_i, i = 1, \ldots, 4\}$ of $\mathbb{R}^4$, orthogonal or not, whereas in the case of general relativity a vierbein is the object to use. In all cases, the definition of a frame includes the law of transformation of the vectors e_i under the appropriate relativity group, such as the Galilei or Poincaré group, or a group of general coordinate transformations. What is it now that plays an equivalent role in quantum physics ? Here the state space of the system is the projective space of a separable, complex, Hilbert space $\mathcal{H}$; observables are represented by self-adjoint operators on $\mathcal{H}$ and the usual probabilistic interpretation applies. What is a quantum frame in this context ?

The simplest candidate is an orthonormal basis $\{\psi_i\}$ in $\mathcal{H}$, giving a resolution of the identity:

$$I = \sum_i |\psi_i\rangle \langle\psi_i|. \tag{1.1}$$

The choice of such a frame is an operational problem, namely it is linked to the choice of some observables that constitute a complete (commuting) set. For simplicity we shall assume that the frame is related to a single observable A, with discrete

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