# FROM QUANTUM MECHANICS TO CLASSICAL MECHANICS AND BACK, VIA COHERENT STATES


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## 1. INTRODUCTION

The coherent states have been introduced in one of the three equivalent ways: i) eigenstates of the harmonic-oscillator operator; ii) action of the displacement operator on the vacuum of the harmonic-oscillator; iii) intelligent states, i.e. quantum states which saturate the position-momentum uncertainty relationship of Heisenberg.[1] Besides, the coherent states have the property: *P) if the initial state is a coherent state and the Hamiltonian is linear in the generators of the group, then the state will evolve into a coherent state.*[2]

Perelomov's definition of coherent state manifold,[3] as orbit of a given group $G$ through a fixed point of the projective Hilbert space $\mathbf{PK}$ attached to the Hilbert space $\mathbf{K}$, generalizes ii) from the Weyl group to arbitrary Lie groups. Perelomov's definition can be globalized.[4] Despite a great number of successful applications,[1,2,5] the exact role of coherent states as a bridge between quantum mechanics and classical mechanics is not completely clear. In this context, an observation which relates the geodesic flow on symmetric spaces $G/K$ to his image in the coherent state manifold $\widetilde{\mathbf{M}}$ is pointed out. Starting with a given quantum mechanical problem, a procedure to associate to a quantum dynamical system a classical one using the coherent states was proposed by Berezin.[6] In some situations,[7] the dequantization is compatible[8,9] with the geometric quantization.[10] The problem of reconstruction of the solution of the initial quantum problem (requantization) consists mainly in finding all unitary equivalence classes of irreducible unitary representations of $G$.[10,11] For Hermitian symmetric spaces,[12–14] this construction is equivalent to geometric quantization.[8,15] For a large class of linear systems, property *P)* (i.e. *"once a coherent state, always a coherent state"*) is still valid.[16] More precisely, for linear dynamical systems, the solution of the Schrödinger equation can be expressed[17] by the coherent vector times the exponential of the sum of the


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