HAMILTONIAN DYNAMICS OF MASSLESS OBJECTS

Andreas Bette

Botvidsgymnasiet, Fågelviksvägen 1B
S-145 53 Norsborg, Sweden.

Abstract

A relativistic dynamics describing a massless particle acted upon by an arbitrary external force is formulated. It turns out that for conservative and central forces, the obtained "massless" equations of motion, are completely integrable. For the value of the helicity equal to zero, hamiltonian flows on the twistor phase space T are constructed which, for conservative forces, reproduce the "massless" equations of motion. A possible generalization of the hamiltonian formalism which is valid for non-vanishing values of the helicity shows that such massless spinning particles should be regarded as one-dimensional extended objects.

1. NOTATION

Latin letters with lower case Latin indices denote four-vectors and four-tensors. Latin letters with lower case Greek indices within brackets denote three-vectors.

In Section 1 the usual three-vector notation (with a bar over a letter) will also be used. In Section 2, on the other hand, a bar over a letter or over an expression denotes complex conjugation.

Lower case Greek letters with upper case Latin indices (either primed or unprimed) denote spinors. Upper case Latin letters with lower case Greek indices denote twistors.

The physical units are so chosen that $c = \hbar = 1$. The signature of the metric g_{ij} in Minkowski space is taken to be $+ - - -$. The fully antisymmetric alternating four-tensor will be denoted by η_{ijkl} with $\eta_{0123} = 1$. The fully antisymmetric alternating three-tensor will be denoted by $\epsilon_{(\alpha)(\beta)(\gamma)}$ with $\epsilon_{(1)(2)(3)} = 1$.

The usual summation convention over repeted indices will be assumed throughout.

2. INTRODUCTION

In order to describe the relativistic classical dynamics of a massless particle (object) as a canonical flow on the twistor phase space we must first analyse the special relativistic version of Newton's second law of dynamics in general and its massless limit in particular.

Newton's second law of dynamics in the Poincaré covariant form:

$$\frac{dY^i}{d\tau} = \frac{P^i}{m}, \tag{1.1}$$

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