STOCHASTIC ASPECTS OF THE QUANTUM DYNAMICS

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Abstract

Following Stratonovich, we make a general analysis of the external force manifestations in the dynamics of Markov diffusion processes. Examples of the standard Brownian motion (Zambrini's "Euclidean quantum mechanics" included) and specific Nelson diffusions are given as an illustration of the formalism.

Let us consider[1,2] a Markovian diffusion $X(t)$ in R^1 (space dimension one is chosen for simplicity) confined to the time interval $t \in [0,T]$, with the point of origin $X(0) = x_0$. The individual (most likely, sample) particle dynamics is symbolically encoded in the Itô stochastic differential equation, which we choose in the form:

$$dX(t) = b(X(t),t)dt + \sqrt{2D}\, dW(t) \tag{1}$$

with $X(0) = x_0, D$ a diffusion coefficient, $W(t)$ a normalised Wiener noise, and the drift field $b(x,t)$ is assumed to guarantee the existence and uniqueness of solutions $X(t)$. They are then non-explosive, i.e. the sample paths of the process cannot escape to spatial infinity in a finite time. The rules of Itô stochastic calculus imply that the transition probability density of the process (its law of random displacements) $p(y,s,x,t), s \leq t$ solves the Fokker-Planck equation with respect to x,t

$$\begin{aligned} \partial_t p &= D\triangle_x p - \nabla_x(bp) \\ \lim_{t\to s} p(y,s,x,t) &= \delta(x-y) \qquad s \leq t \end{aligned} \tag{2}$$

Following Stratonovich,[3] let us transform (2) by means of a substitution

$$p(y,s,x,t) = h(y,s,x,t)\frac{\exp\Phi(y,s)}{\exp\Phi(x,t)} \tag{3}$$

which under an assumption that $b(x,t)$ is the gradient field

$$b(x,t) = -2D\nabla\Phi(x,t) \Rightarrow \frac{1}{2}[\frac{b^2}{2D} + \nabla b] = D[(\nabla\Phi)^2 - \triangle\Phi] \tag{4}$$