SUPERCOHERENT STATES AND GEOMETRIC QUANTIZATION OF A SUPER KÄHLER SUPERMANIFOLD

Amine M. El Gradechi[1,2]

[1] Centre de Recherches Mathématiques, Université de Montréal
C.P. 6128 succursale Centre-ville, Montréal, Canada H3C 3J7

[2] Department of Mathematics and Statistics, Concordia University
Montréal, Canada H4B 1R6

Abstract

The standard group theoretic construction of coherent states has recently been extended to simple Lie supergroups, yielding the so-called supercoherent states. Usual coherent states for semi-simple Lie groups are parameterized by points of a symplectic homogeneous space, which is moreover a Kähler manifold. Analogously, we show here that the OSp(1/2) coherent states are parameterized by an OSp(1/2) *supersymplectic homogeneous superspace*. This turns out to be a non-trivial example of Rothstein's general supersymplectic supermanifolds, and leads to the definition of the notion of a *super Kähler supermanifold*. This new subcategory of supermanifolds is well suited for the super extension of geometric quantization. Indeed, super Kähler supermanifolds are naturally equipped with a *super Kähler polarization*. The full geometric quantization procedure is here extended to the super Kähler homogeneous superspace underlying the OSp(1/2) coherent states. The present talk is based on results obtained in Refs. 1 and 2.

1. INTRODUCTION

Classical and quantum theories describing G-elementary systems, for a given Lie group G, are well understood theories. Classical G-elementary systems are described by coadjoint orbits of G, which are homogeneous symplectic spaces for G. Namely, these are phase spaces $(G/H, \omega)$, where H is a closed subgroup of G and ω is a G-invariant, closed and non-degenerate two-form on G/H. Quantum G-elementary systems are described by a pair $(U(G), \mathcal{H})$, where $U(G)$ is a Unitary Irreducible Representation (UIR) of G in some Hilbert space $\mathcal{H}$.

Classical and quantum G-elementary systems can generally be related to each other through known methods. These are, on the one hand, the quantization methods, such as geometric quantization, and on the other hand, the classical limit procedures. From a physical point of view the latter are the most natural paths. However, within their limits of application, quantization methods provide mathematical means of translating the well established understanding of the classical theories to the quantum realm.

It happens that one knows UIRs of some Lie group G and wants then to look for the classical theories they are associated to. This classical limit can be reached using

Quantization and Infinite-Dimensional Systems
Edited by J-P. Antoine *et al.*, Plenum Press, New York, 1994