TWO APPROACHES TO TOEPLITZ OPERATORS ON FOCK SPACE

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1. INTRODUCTION

The concrete model of the Fock space as an L^2 -space of entire functions established by Segal[10] and developed by Bargmann[1] provides a convenient and mathematically precise language for studying free Bose fields. C. Berger and L. Coburn[2,3] (using earlier ideas of F.A. Berezin and W. Arveson) have proposed to view a broad family of observables as Toeplitz operators. This allows to unify the operator–theoretic study, linking it with function theory via symbol analysis. Related results for special types of analytic symbols can be traced much earlier in a paper by Newman with Shapiro.[9]

The intense development of the theory of Toeplitz operators (related to many objects) is also taking place in the setting of the Segal–Bargmann space, but the results are confined, in principle, to the spaces of functions depending on finitely many variables. Especially, the case of unbounded symbols, which still offers more open questions than answers.[5,7]

Our attempts to carry at least a part of this program for infinitely many variables (degrees of freedom) have met serious difficulties.[6] The basic one is related to the nonuniqueness of the CCR's: we have to choose between non-equivalent models, i.e., between non-equivalent definitions of the Toeplitz operator T_φ corresponding to the given symbol φ .

Here we want to compare the two approaches that seem most natural from the mathematical point of view.

2. THE MEASURE – THEORETIC APPROACH

If one prefers the natural way of defining Toeplitz operators, two objects are needed: a measure space structure (Ω, Σ, μ) over some set Ω and a closed subspace $\mathcal{B}$ of $L^2(\mu)$, usually consisting of functions holomorphic on Ω. Let $P : L^2(\mu) \to \mathcal{B}$ be the orthogonal projection. Given a measurable function φ on Ω , one defines the operator

$$T_\varphi f = P(\varphi f), \tag{2.1}$$

with the domain

$$DT_\varphi = \{f;\ (\varphi f) \in L^2(\mu)\}. \tag{2.2}$$

We want to apply this scheme in the case where $\mathcal{B}$ is the Segal - Bargmann space $\mathcal{F}$ of functions on a separable complex Hilbert space H. The chief advantages of this

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