

SYMPLECTIC REALIZATIONS OF THE GALILEI-CARROLL GROUP

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Abstract

We generalize the Galilei-Carroll group $G^{1,2}$ we classify all his G -elementary systems under the coadjoint action of G on the dual \mathcal{H}^* of the Lie algebra \mathcal{H} , the central extension of the Lie algebra of G .

1. THE GALILEI-CARROLL GROUP

Let M be a $(n + 1)$ -dimensional space endowed with the metric

$$\eta_{\mu\nu} = \text{diag}(0, -1, \dots, -1). \quad (1)$$

Let $x^\mu = \begin{pmatrix} \xi \\ x^i \end{pmatrix}$, $i = 1, \dots, n$, be the coordinates of an arbitrary point of M . Then the displacement group associated with (1) is the group of transformations

$$x_2^\mu = g_\nu^\mu x_1^\nu + x^\mu, \quad (2)$$

where

$$x^\mu = \begin{pmatrix} \xi \\ x^i \end{pmatrix}, \quad (g_\nu^\mu) = \begin{pmatrix} 1 & u_j R_i^j \\ 0 & R_i^j \end{pmatrix}, \quad (R_i^j) \in SO(n). \quad (3)$$

Under the usually composition law of matrices, one can verify that the group of transformations (2) is exactly the Carroll group³, whose multiplication law is

$$\begin{aligned} & (\xi, x^i, u_i, R_k^i) (\xi', x'^k, u'_k, R'_j{}^k) \\ & = (\xi + u_i R_k^i x'^k + \xi', x^i + R_k^i x'^k, u_i + R_i^k u'_k, R_k^i R'_j{}^k). \end{aligned} \quad (4)$$

Now, let $V = M \times \mathbb{R}$ be a $(n+1)$ -dimensional space, where \mathbb{R} supports the absolute time coordinate. Let $x^a = \begin{pmatrix} x^\mu \\ \xi \end{pmatrix}$ be the coordinates of an element in V . Then one verifies that the isotropy and homogeneity of V together with the galilean principle of relativity of motion,² the isotropy and homogeneity of time, admit only the transformations

$$\begin{aligned} x_2^\mu &= g_\nu^\mu x_1^\nu + v^\mu t_1 + x^\mu \\ t_2 &= t_1 + t, \end{aligned} \quad (5)$$