ON REPRODUCING KERNELS FOR HOLOMORPHIC VECTOR BUNDLES

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Abstract

We introduce (1) the reproducing kernels of Bergman type for holomorphic sections of complex hermitian vector bundles, and (2) the maps defined by these kernels on total and base spaces of considered bundles into some Hilbert and Grassmann spaces. We present (without proofs) the main results concerning basic properties of the introduced objects.

The purpose of this paper is to present the main results concerning reproducing kernels of the Bergman function type for holomorphic sections of complex vector bundles. Most of the proofs and some other details can be found in Refs. 1-3.

Assume that there are given:

$\mathbf{E} = (E, \pi, M)$ -a holomorphic vector bundle over a complex manifod M;

$\mu \in \Gamma^\infty(\bigwedge^{2n} T^*M)$ -a volume form on M, where $n := dim_{\mathbf{C}} M$;

h -a hermitian structure on $\mathbf{E}$.

We denote by $L^2(\mathbf{E}) = L^2(\mathbf{E}, h, \mu)$ the Hilbert space obtained by completion of the space $\Gamma_0^\infty(\mathbf{E})$ of all smooth sections of $\mathbf{E}$ with compact support with respect to the norm $\|\cdot\|$ defined by the scalar product

$$< s \mid t > := \int_M h(s,t)\mu, \qquad s,t \in \Gamma_0^\infty(\mathbf{E}).$$

It is known that $L^2(\mathbf{E})$ can be identified with the space (of classes) of all Lebesgue measurable sections s of $\mathbf{E}$ for which the integral

$$\|s\|^2 = \int_M h(s,s)\mu(s)$$

is finite. When M has a countable basis of topology one can prove that $L^2(\mathbf{E})$ is a separable Hilbert space.

Let $L^2H(\mathbf{E}) = L^2H(\mathbf{E}, h, \mu)$ denote the space of all elements of $L^2(\mathbf{E})$ which can be identified with holomorphic sections of $\mathbf{E}$ i.e.

$$L^2H(\mathbf{E}) = L^2(\mathbf{E}) \cap \mathcal{O}(M, \mathbf{E}),$$

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