POISSON STRUCTURES, STABILITY AND CONTROL

Mircea Puta

Seminarul de Geometrie-Topologie
University of Timişoara, 1900 Timişoara
România

Abstract

The theory of Hamilton-Poisson mechanical systems is discussed and some of its applications in nonlinear stability and control theory are pointed out.

1. INTRODUCTION

New analytical techniques and recent algorithms which numerically compute the time evolution of mechanical systems enable today's scientists, engineers and mathematicians to predict events more accurately and more rapidly than ever before. Beyond the problems of simulation and prediction, however, lies the problem of understanding the mathematical modelling of a given physical situation. Many systems remain too intricate to fully understand, but modern methods of differential geometry can sometimes offer insights. Most of these insights are obtained by viewing dynamics geometrically, and in fact the recent advances in mechanics all share this geometric perspective.

We shall try here to discuss in detail some geometrical aspects of a particular class of mechanical systems, the so called Hamilton-Poisson mechanical systems.

2. HAMILTON-POISSON MECHANICAL SYSTEMS

Let us start with a brief introduction to the Hamiltonian mechanics.

Definition 2.1 – A Hamiltonian mechanical system is a triple (M, ω, H) where (M, ω) is a $2n$-dimensional symplectic manifold and H is a smooth real valued function defined on M and called the energy or the Hamiltonian of the system.

The evolution in time of a Hamiltonian mechanical system is given by the integral curves of the Hamiltonian vector field X_H, where X_H is uniquely determined by the condition

$$i_{X_H}\omega + dH = 0,$$

or equivalent by Hamilton's equations:

$$\dot{q}^i = \frac{\partial H}{\partial p_i},$$

Quantization and Infinite-Dimensional Systems
Edited by J-P. Antoine *et al.*, Plenum Press, New York, 1994