RICCATI EQUATION OVER TORUS AND SEMICLASSICAL QUANTIZATION OF MULTIPERIODIC MOTION

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Abstract

The semiclassical wave functions and the eigenvalues of the 3-dimensional quantum systems associated with an invariant 2-torus are constructed in terms of solutions of the Riccati equation and certain universal Gaussian packets over the torus. The quantization rule of the Bohr-Sommerfeld type is discussed which gives the exact spectrum for certain integrable systems.

0. INTRODUCTION

In this paper we discuss some aspects of semiclassical quantization of invariant tori of lower dimension. This problem is studied from different points of view, for example, in Refs. 1–5. We shall use the integral representation of wave functions recently proposed in Refs. 6–8 and its isotropic version.[9–11] This representation is based on a linear superposition (an integration) of certain universal Gaussian packets associated with an invariant torus. The idea of integrating the Gaussian packets along classical trajectories was first proposed in Ref. 12 for certain concrete examples. We consider the following situation.

Suppose that a Hamiltonian system corresponding to the function $H = H(p,q)$ on the 6-dimesional phase space $\mathcal{X} = \mathbf{R}^3_p \oplus \mathbf{R}^3_q$ with symplectic structure $\Omega = dp \wedge dq$ has an *isotropic invariant 2-dimensional torus* $\Lambda \approx \mathbf{T}^2$:

$$\Lambda \subset \mathcal{X}, \qquad \Omega|_\Lambda = 0, \qquad \dim \Lambda = 2$$

with quasiperiodic motion

$$\mathrm{ad}(H)|_\Lambda = \omega \cdot \frac{\partial}{\partial \alpha} \equiv \sum_{j=1}^{2} \omega_j \frac{\partial}{\partial \alpha_j}, \qquad \omega_j = \mathrm{const},$$

where $\mathrm{ad}(H)$ is a Hamiltonian field associated with H, $\alpha = (\alpha_1, \alpha_2)$ $(0 \le \alpha_j \le 2\pi)$ are the cyclic coordinates on Λ and $\omega = (\omega_1, \omega_2)$ is a frequency vector. Assume also that

$$\left.\frac{\partial H}{\partial p_3}\right|_\Lambda = \left.\frac{\partial H}{\partial q_3}\right|_\Lambda = 0.$$

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