Symmetries of Hamiltonian Systems on Symplectic and Poisson Manifolds

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1 Introduction

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1.1 Contents of the Paper

Symplectic and Poisson manifolds are defined in Sects. 2 and 3, where their basic properties are given, often with detailed proofs. Darboux theorem and the main results about the local structure of Poisson manifolds, however, are given without proof. Actions of a Lie group or of a Lie algebra on a smooth manifold and, when this manifold is endowed with a symplectic or a Poisson structure, symplectic, Poisson and Hamiltonian actions are introduced in Sect. 4. For Hamiltonian actions of a Lie group on a connected symplectic manifold, the equivariance of the momentum map with respect to an affine action of the group on the dual of its Lie algebra is

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